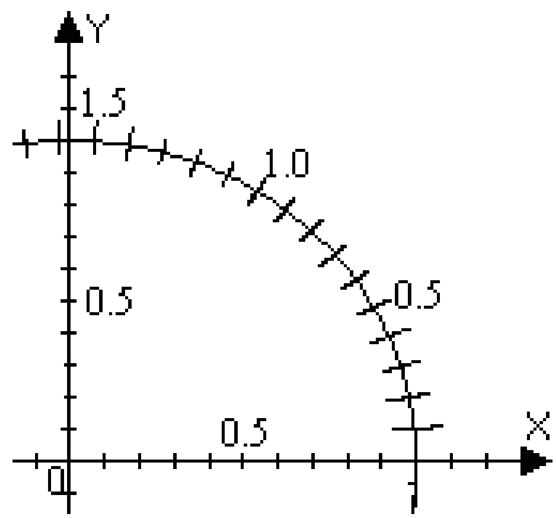
Math 5 – Trigonometry – Chapter 4 Test – spring ’09 Name_________________________

Show your work for credit. Write all responses on separate paper. Don’t use a calculator.

1. If the arclength $t = \dfrac{29\pi}{6}$ is traced counterclockwise along the unit circle from (1,0) then
   a. What is the reference number for t ?
   b. What are the coordinates of the terminal point $P(x,y)$?
   c. Draw the unit circle and plot the terminal point $P(x,y)$.

2. Consider the point $\left(\dfrac{5}{13}, \dfrac{12}{13}\right)$
   a. Verify that the point lies on the unit circle.
   b. Use the diagram at right to approximate to the nearest tenth a value of t so that $\cos(t) = \dfrac{5}{13} \approx 0.38$
   c. Approximate to the nearest tenth the interval in the first quadrant where $\dfrac{5}{12} \le \tan(t) \le \dfrac{12}{5}$



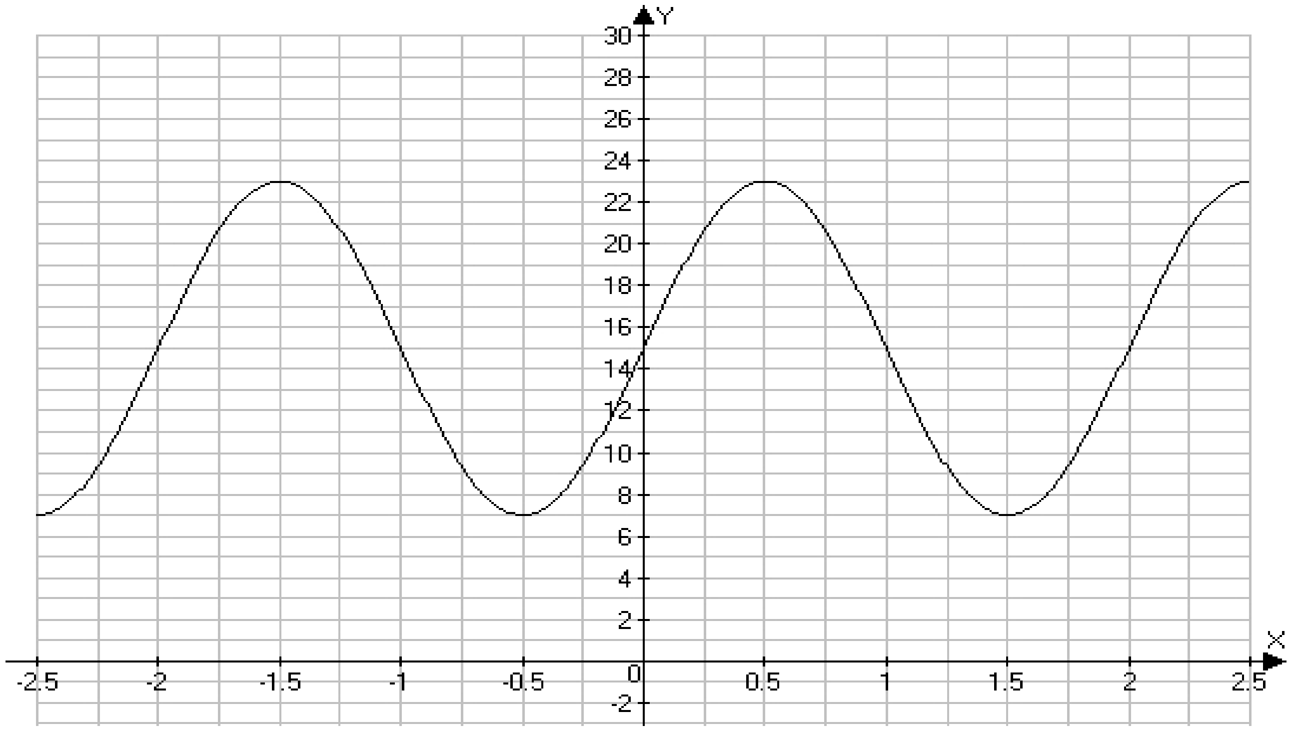
3. Recall that a function is even if $f(-x) = f(x)$ and odd if $f(-x) = -f(x)$. Of the six trigonometric functions: sin(x), cos(x), tan(x), sec(x), csc(x) and cot(x)
   a. Which functions are even?
   b. Which functions are odd?

4. Suppose that $\cos(t) = \dfrac{\sqrt{91}}{100}$ and point and sin(t) < 0.
   Find $\sin(t), \tan(t), \sec(t), \csc(t)$ and $\cot(t)$.

5. Write $\sec(t)$ in terms of $\tan(t)$, assuming the terminal point for t is in quadrant III.

6. Find the amplitude, period and phase shift of $y = 5 + 5\sin\left(20\pi\left(x - \dfrac{1}{50}\right)\right)$, construct a table of values and graph one period of the function, clearly showing the position of key points.

7. Find an equation for the sinusoid whose graph is shown:



8. Consider the function $f(x) = \tan\left(\frac{\pi}{2}x\right)$.
   a. Find the equations for two adjacent vertical asymptotes and sketch them in with dashed lines.
   b. Find the *x*-coordinates where $y = 0$ and where $y = \pm 1$.
   c. Carefully construct a graph of the function showing how it passes through the points where $y = -1$, $y = 0$, $y = 1$ and how it approaches the vertical asymptotes.

9. Suppose $\cos t = 9/28$ and *t* is in the first quadrant. Find the following:
   a. $\cos(t + \pi)$
   b. $\cos\left(t + \frac{\pi}{2}\right)$
   c. $\cos\left(\frac{\pi}{2} - t\right)$

10. The Millennium Wheel rotates once every 30 minutes. Its highest point is about 135 meters above the ground and the lowest point is about 5 meters above the ground. Write a function that gives the height of a rider *t* minutes after boarding the Millennium Wheel.

Math 5 – Trigonometry – Chapter 4 Test Solutions – fall ’07

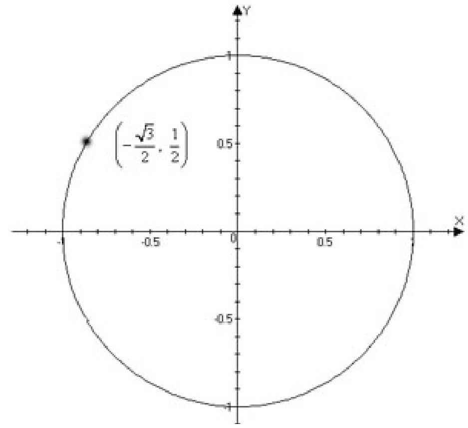
1. For arclength $t = \dfrac{29\pi}{6}$ traced counterclockwise along the unit circle from (1,0)

   a. Find the reference number for *t*.

   SOLN: $t = \dfrac{29\pi}{6} = \dfrac{(12+12+5)\pi}{6} = 2\pi + 2\pi + \dfrac{5\pi}{6}$ so the reference number is $\dfrac{\pi}{6}$.

   b. What are the coordinates of the terminal point $P(x,y)$?
   SOLN: Since this point is in the 2nd quadrant, *x* is negative, but $y > 0$. $x = -\cos\left(\dfrac{\pi}{6}\right) = -\dfrac{\sqrt{3}}{2}$ and $y = \sin\left(\dfrac{\pi}{6}\right) = \dfrac{1}{2}$.

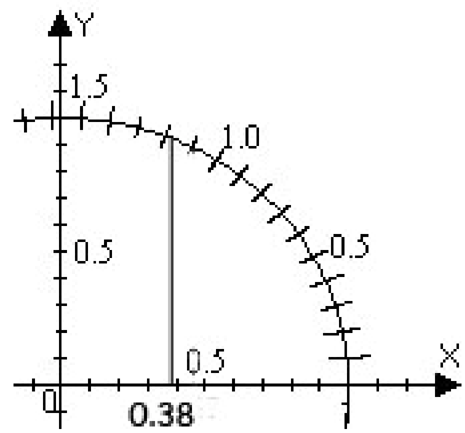
   c. Illustrate this point’s position on a plot of the unit circle.
   ANS: The point $\left(-\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right)$ is shown



2. Consider the point $\left(\dfrac{5}{13}, \dfrac{12}{13}\right)$

   a. Verify that the point lies on the unit circle.
   ANS: $\left(\dfrac{5}{13}\right)^2 + \left(\dfrac{12}{13}\right)^2 = \dfrac{25}{169} + \dfrac{144}{169} = \dfrac{169}{169} = 1$

   b. Use the diagram at right to approximate to the nearest tenth a value of *t* so that $\cos(t) = \dfrac{5}{13} \approx 0.38$
   ANS: A vertical segment is drawn from 0.38 on the *x*-axis intersects the circle at *t* between 1.1 and 1.2: closer to *t* = 1.2. Indeed, arccos(5/13) is approximately 1.176



   c. Approximate to the nearest tenth the interval in the first quadrant where $\dfrac{5}{12} \le \tan(t) \le \dfrac{12}{5}$
   ANS: If cos(*t*) = 5/13 and sin(*t*) = 12/13, then tan(*t*) = 12/5. Since cot(*t*) = cos(*t*)/sin(*t*) = 5/12 and tan(π/2 – *t*) = cot(*t*). So choose *t* = 1.6 – 1.2 = 0.4 so that the approximate *t* interval we seek is *t* between 0.4 and 1.2.

3. Recall that a function is even if $f(-x) = f(x)$ and a function is odd if $f(-x) = -f(x)$. Of the six trigonometric functions, which are even and which are odd?
   SOLN: Only cos(*t*) and sec(*t*) are even . The other four are odd.

4. Suppose that $\cos(t) = \dfrac{\sqrt{91}}{100}$ and point and sin(*t*) < 0.
   Find $\sin(t), \tan(t), \sec(t), \csc(t)$ and $\cot(t)$.

   ANS: $\sin(t) = -\sqrt{1-\cos^2 t} = -\sqrt{1-\left(\dfrac{\sqrt{91}}{100}\right)^2} = -\sqrt{1-\dfrac{91}{10000}} = -\sqrt{\dfrac{10000-91}{10000}} = -\sqrt{\dfrac{9909}{10000}} = -\dfrac{3\sqrt{1101}}{100}$

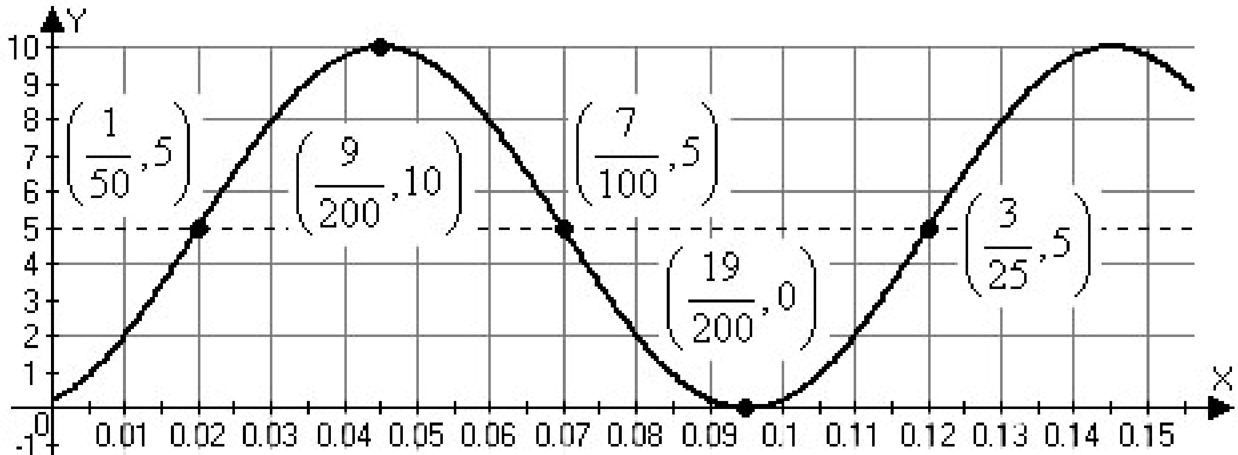
   $\tan(t) = -\dfrac{3\sqrt{1101}}{\sqrt{91}} = -\dfrac{3\sqrt{100191}}{91}$; $\sec(t) = \dfrac{100\sqrt{91}}{91}$; $\csc(t) = -\dfrac{100\sqrt{1101}}{3303}$; $\cot(t) = -\dfrac{\sqrt{100191}}{3303}$

5. Write $\sec(t)$ in terms of $\tan(t)$, assuming the terminal point for t is in quadrant III.

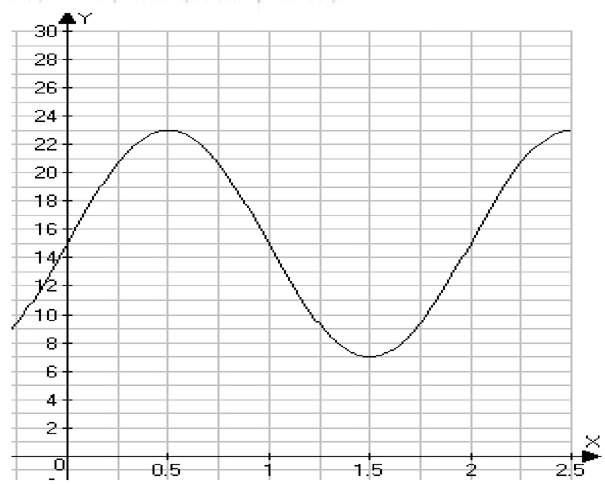
   ANS: Starting with $\cos^2 t+\sin^2 t=1$, divide through by $\cos^2 t$ to obtain $1+\tan^2 t=\sec^2 t$. Since sec(t) is negative in quadrant III, $\sec t=-\sqrt{1+\tan^2 t}$

6. Find the amplitude, period and phase shift of $y=5+5\sin\left(20\pi\left(x-\frac{1}{50}\right)\right)$, construct a table of values and graph one period of the function, clearly showing the position of key points.

   ANS: The amplitude is 5, the period is 1/10 and the phase angle is 1/50.
   Graph is shown below. The starting point and endpoint are $\left(\frac{1}{50},5\right)$ and $\left(\frac{3}{25},5\right)$. The halfway point is $\left(\frac{7}{100},5\right)$ while the peak and trough are, respectively, $\left(\frac{9}{200},10\right)$ and $\left(\frac{19}{200},0\right)$.



7. Find an equation for the sinusoid whose graph is shown:



   ANS: The lowest point is at y=7 and the highest point is at 23 so the line of equilibrium is at the average of these: y = (7+23)/2 = 15. and the amplitude is (23 – 7)/2 = 8.

   The two peaks shown in the graph here are where x = 0.5 and x = 2.5, so the period is 2.5 – 0.5 = 2.
   Thus an equation for the sinusoid is $y=15+8\sin(\pi x)$.

8. Consider the function $f(x)=\tan\left(\frac{\pi}{2}x\right)$.

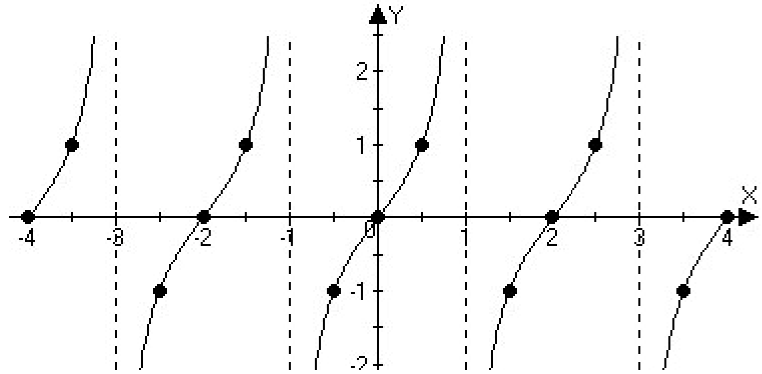
   a. Find the equations for two adjacent vertical asymptotes and sketch them in with dashed lines.

      ANS: We want the input to the tangent to be $\pm\frac{\pi}{2}$, that is $\frac{\pi}{2}x=\pm\frac{\pi}{2}\Leftrightarrow x=\pm1\Leftrightarrow \boxed{x=-1 \text{ or } 1}$

b. Find x-coords where $y = 0$ and $y = \pm 1$.
SOLN: If x is any even integer then $f(2k) = \tan(k\pi) = 0$, which is true for any integer value for k.

Also We want to find where the input to the tangent function is equal to $\pm\frac{\pi}{4}$, that is

$$\frac{\pi}{2}x = \pm\frac{\pi}{4} \Leftrightarrow x = \pm\frac{1}{2}$$



c. The pattern of even integer $x = 2k$ intercepts and points: $(2k–1/2, –1)$ and $(2k+1/2, 1)$ is evident in the graph. Also note that $x = 2k + 1$ are vertical asymptotes for $k \in \mathbb{Z}$

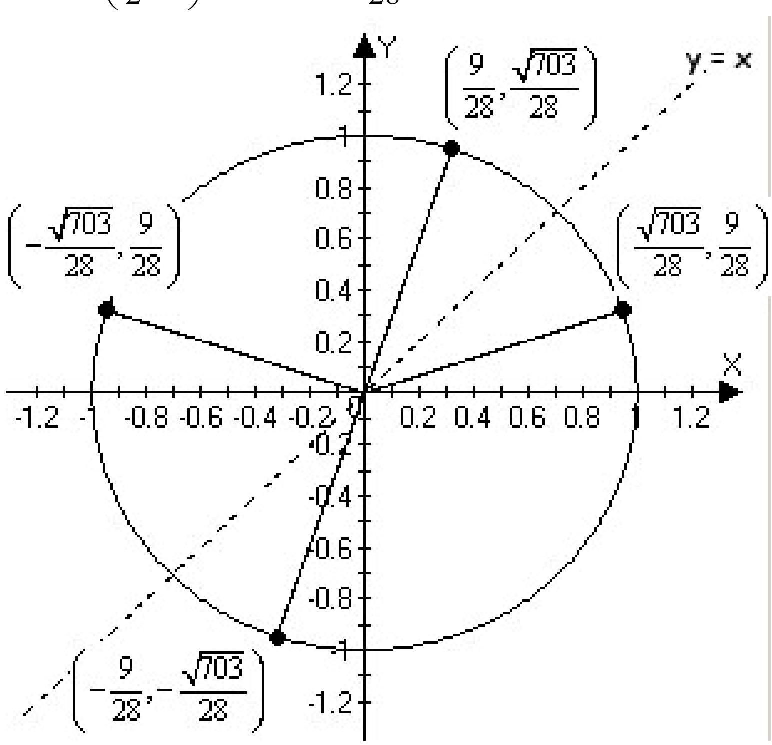
9. Suppose $\sin t = 9/28$ and t is in the first quadrant. Find the following:

a. $\cos(t+\pi) = -\dfrac{9}{28}$

b. $\cos\left(t+\dfrac{\pi}{2}\right) = \sin(t) = \sqrt{1-\left(\dfrac{9}{28}\right)^2} = \sqrt{1-\dfrac{81}{784}} = \sqrt{\dfrac{703.}{784}} = \dfrac{\sqrt{703.}}{28}$

Note that 703 = 19*37 is not prime but is square free.

c. $\cos\left(\dfrac{\pi}{2}-t\right) = \sin(t) = \dfrac{\sqrt{703}}{28}$



10. The Millennium Wheel rotates once every 30 minutes. Its highest point is about 135 meters above the ground and the lowest point is about 5 meters above the ground. Write a function that gives the height of a rider t minutes after boarding the Millennium Wheel.
ANS: $h(t) = 70 - 65\cos(\pi t/15)$